Name of the Institute	**:**	**C.K.Pithawalla College of Engineering & Technology, Surat**
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Contents

1. First Order Ordinary Differential Equation.

1.1 (May 2012)

1. Solve the differential equation $xy\frac{dy}{dx} = 1 + x + y + xy$. (1)
2. Solve $(x+y)^2\left[x\frac{dy}{dx} + y\right] = xy\left[1 + \frac{dy}{dx}\right]$. (2)
3. Solve the differential equation : $\left(x^2y^2 + 2\right)ydx + \left(2 - x^2y^2\right)xdy = 0$. (3)

1.2 (December 2011)

4. Solve : $9yy' + 4x = 0$. (2)
5. Solve the Bernoulli equation $y' + y\sin x = e^{\cos x}$. (3)
6. Solve the IVP : $xy' + y = 0; y(2) = -2$. (2)
7. Test for exactness and solve : $\left[(x+y)e^x - e^y\right]dx - xe^y dy = 0; y(1) = 0$. (3)

1.3 (May 2011)

8. Find the order and degree of the differential equation $\left[\frac{dy}{dx} + y\right]^{\frac{1}{2}} = \sin x$. (1)
9. Solve $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$. (3)

1.4 (December 2010)

10. Solve : $xy' = y^2 + y$. (2)
11. Solve the initial value problem $y' - \left(1 + 3x^{-1}\right)y = x + 2; y(1) = e - 1$. (3)
12. Find the orthogonal trajectories of the curve $y = x^2 + c$. (3)
13. Solve : $y' + \frac{1}{3}y = \frac{1}{3}(1 - 2x)x^4$. (3)
14. Solve the IVP $L\frac{dI}{dt} + RI = 0; I(0) = I_0$, where R, L & I_0 being constants. (3)

1.5 (March 2010)

15. Find the solution of differential equation $ye^x dx + \left(2y + e^x\right)dy = 0$, where $y(0) = -1$. (2)

16. Solve the differential equation $y' + y\sin x = e^{\cos x}$. (3)

17. Solve the differential equation $y' + 6x^2 y = \frac{e^{-2x^3}}{x^2}$, where $y(1) = 0$. (3)

1.6 (December 2009)

18. Solve : $\frac{dy}{dx} + y = x$. (2)

19. Solve the following differential equations:

  a. $2xydx + x^2dy = 0$ (2)

  b. $\frac{dy}{dx} - y = e^{2x}$. (2)

  c. $\frac{dy}{dx} + y = -\frac{x}{y}$. (3)

2. Higher Order Linear ODE's with constant coefficient.

2.1 (May 2012)

20. Find the general solution of $\frac{d^4y}{dx^4} - 18\frac{d^2y}{dx^2} + 81y = 0$. (1)

21. Find the particular solution of $y = \frac{1}{(D+1)^2}\cosh x,\ where\ D = \frac{d}{dx}$. (1)

22. Find the solution of $y'' - 5y' + 6y = 0$ with initial condition $y(1) = e^2$ & $y'(1) = 3e^2$. (2)

23. Solve : $\frac{d^4y}{dt^4} - 2\frac{d^2y}{dt^2} + y = \cos t + e^{2t} + e^t$. (3)

24. Solve : $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = \frac{e^{2x}}{x^5}$. (3)

25. Solve : $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 5y = e^x\cos 3x$. (3)

26. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2e^x\cos\left(\frac{x}{2}\right)$. (3)

27. Solve : $\frac{d^3y}{dx^3} + \frac{dy}{dx} = \operatorname{cosec} x$ by method of variation of parameters. (3)

28. Solve : $\left(D^2 - 4D + 4\right)y = \frac{e^{2x}}{1+x^2}, where\ D = \frac{d}{dx}$. (3)

29. **(December 2011)**

30. Verify that $y=e^{x}\left(a\cos x+b\sin x\right)$ is a solution of $y''+2y'+2y=0$. (2)

31. Verify that the functions $x^{-\frac{1}{2}}$ & $x^{\frac{3}{2}}$ form a basis of solutions of $4x^2y''-3y=0$ and solve it when $y(1)=3, y'(1)=2.5$. (3)

32. Find the general solution of $\left(D^2+1\right)y=0$. (2)

33. Do as directed:

    a. Solve the IVP : $xy'+y=0; y(2)=-2.$ (2)

    b. Solve : $y'+y\sin x=e^{\cos x}$. (3)

    c. Solve : $y''+4y'+4y=0, y(0)=1, y'(0)=1$. (2)

34. Find the general solution : $16y''-8y'+5y=0$. (2)

35. Solve the non-homogeneous equations (7)

    a. $y''-3y'+2y=e^{x}$.

    b. $y''+y=\sec x$.

36. Do as directed:

    a. Find the general solution of : $y'''-y''+100y'-y=4e^{t}$. (4)

    b. Solve : $y'''-y''+100y'-100y=0; y(0)=4, y'(0)=11, y''(0)=-299.$ (3)

2.2 (May 2011)

37. Write Abel-Liouville formula. Use it to check that the set $\{x, x^2, x\log|x|\}$ is a basis for some third order linear ordinary differential equation. (2)

38. Solve : $\left(D^2+a^2\right)y=\cos ec\ ax$. (4)

39. Obtain the second linearly independent solution of $xy''+2y'+xy=0$ given that $y_1(x)=\dfrac{\sin x}{x}$ is one solution. (3)

40. Solve : $\left(D^4+2a^2D^2+a^4\right)y=\cos ax$. (4)

41. Solve the initial value problem by method of undetermined coefficients $y''+3y'+y=30e^{-x}, y(0)=3, y'(0)=-3, y''(0)=-47$. (4)

2.3 (December 2010)

42. Find the second order homogeneous linear differential equation for which the functions $x, x^2\log x$ are solutions. (2)

43. Solve : $y'' + 2y' + 2y = 0; y(0) = 1, y\left(\frac{\pi}{2}\right) = 0$. (2)
44. Using the method of variation of parameters, find the general solution of the differential equation $\left(D^2 - 2D + 1\right)y = 3x^{\frac{3}{2}}e^x$. (5)
45. Find a basis of solution for the differential equation $x^2y'' - xy' + y = 0$, if one of its solutions is $y_1 = x$. (3)
46. Using the method of undetermined coefficient, find the general solution of the differential equation $y'' + 2y' + 10y = 25x^2 + 3$. (4)
47. Solve the IVP : $y'' + 4y = 8e^{-2x} + 4x^2 + 2; y(0) = 2, y'(0) = 2$. (5)

2.4 (March 2010)

48. Find the solution of differential equation $y'' + 4y = 2\sin 3x$ by method of undetermined coefficient. (2)
49. Find the particular solution of $y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x.$ (2)
50. Find the general solution of $y'' + 9y = \sec 3x$ by method of variation of parameter. (3)
51. Solve $\left(D^2 + 4D + 4\right)y = \frac{e^{-2x}}{x^2}$. (3)

2.5 (December 2009)

52. Solve the IVP : $y'' + y' - 2y = 0; y(0) = 4, y'(0) = -5$. (5)
53. Given the functions e^x & e^{-x} on any interval $[a,b]$. Are these functions L.I. or L.D.? (4)
54. Using the method of variation of parameter solve $y'' + y = \sec x$. (5)
55. Using the method of undetermined coefficients, solve $y'' + 4y = 8x^2$. (5)
56. Find the steady state oscillation of the mass-spring system governed by the equation $y'' + 3y' + 2y = 20\cos 2t$. (5)

3. Cauchy Euler Equations

3.1 (May 2012)

57. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\left(\log(1+x)\right)$. (3)
58. Solve : $x^2D^2 - 3xD + 4 = x^2$ given that $y(1) = 1$ & $y'(1) = 0.$ (3)

3.2 (December 2011)

59. Solve : $\left(x^2D^2-3xD+4\right)y=0; y(1)=0, y'(1)=3$. (2)

3.3 (May 2011)

60. Solve : $x^2y''-2.5xy'-2.0y=0$. (2)

61. Solve $x^3y'''+2x^2y''+2y=10\left(x+\frac{1}{x}\right)$. (4)

62. Solve : $x^2y''-4xy'+6y=21x^{-4}$. (4)

63. Solve the non-homogeneous Euler-Cauchy equation $x^3y'''-3x^2y''+6xy'+6y=x^4\log x$ by variation of parameters method. (4)

3.4 (December 2010)

64. Find the general solution of the equation $\left(x^2D^2-2xD+2\right)y=x^3\cos x$. (4)

3.5 (March 2010)

65. Solve $\left(x^2D^2-3xD+3\right)y=3\ln x-4$. (3)

3.6 (December 2009)

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4. Beta and Gamma Functions.

4.1 (May 2012)

66. Find the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$. (1)

67. Evaluate $\int_0^1 x^4\cos^{-1}x\,dx$. (1)

68. Evaluate $\int_0^7 (x-3)^{\frac{1}{4}}(7-x)^{\frac{1}{4}}\,dx$. (2)

69. Evaluate $\int_0^1 (x\log x)^3\,dx$. (2)

4.2 (December 2011)

70. Show that $\Gamma(m+1)m!$, where Γ is the Gamma function and m is positive integer. (3)

71. Using the Beta and Gamma functions evaluate the integral $\int_{-1}^{1}\left(1-x^2\right)^n dx$, where n is a positive integer. (3)

4.3 (May 2011)

72. Write duplication formula. Use it to find the value of $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$. (2)

73. Prove that $\frac{B(m+1,n)}{B(m,n)}=\frac{m}{m+n}$. (2)

4.4 (December 2010)

74. Evaluate : $\int_{0}^{1} x^4\left[\log\left(\frac{1}{x}\right)\right]^3 dx$. (2)

75. Compute $B\left(\frac{9}{2},\frac{7}{2}\right)$. (2)

76. Prove that $\int_{0}^{1}\frac{x}{\sqrt{1-x^5}}dx=\frac{1}{5}B\left(\frac{2}{5},\frac{1}{2}\right)$. (3)

4.5 (March 2010)

77. Evaluate $\int_{0}^{\infty} x^m e^{-ax^n} dx$. (2)

78. Evaluate $\int_{-1}^{1}(1+x)^m(1-x)^n dx$, where $m>0, n>0$ are integers. (3)

4.6 (December 2009)

79. Evaluate the integral $\int_{0}^{\infty}\exp\left(-x^2\right)dx$. (2)

80. Using the relationship between the beta and gamma functions, simplify the expression $B(m,n)B(m+n,p)B(m+n+p,q)$. (2)

81. Express $\int_{0}^{1} x^m\left(1-x^n\right)^p dx$ in terms of Gamma function. (2)

82. State Legendre duplication formula. Hence prove that $B(m,m)B\left(m+\frac{1}{2},m+\frac{1}{2}\right)=\pi m^{-1}2^{1-4m}$. (3)

5. Laplace Transforms.

5.1 (May 2012)

83. Find the Laplace transform of $f(t) = t^2 \sinh at$. (2)

84. Find the Laplace transform of $f(t) = \begin{cases} 0; 0 < t < \pi \\ \sin t; t > \pi \end{cases}$. (2)

85. Find the inverse Laplace transform of $\frac{5s+3}{(s^2+2s+5)(s-1)}$. (3)

86. Find the Laplace transform of $\frac{1-\cos t}{t}$. (2)

87. Using Laplace transform solve the differential equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t}\sin t$ where, $x(0) = 0 \;\&\; x'(0) = 1$. (3)

88. Find the Laplace transform of (3)
    a. $e^{-3t}u(t-2)$
    b. $\int_0^t e^{-u}\cos u\,du$.

89. Solve the differential equation $\frac{d^2y}{dt^2} + 4y = f(t), y(0) = 0, y'(0) = 1$ by Laplace transform
    a. $f(t) = \begin{cases} 1; 0 < t < 1 \\ 0; t > 1 \end{cases}$
    b. $f(t) = H(t-2)$. (03)

5.2 (December 2011)

90. Find the Laplace transform of $f(t) = \begin{cases} 0; 0 \le t < 2 \\ 3; t \ge 2 \end{cases}$. (2)

91. Define the terms : Laplace transform of $f(t)$, and its Inverse transform. (2)

92. Find the Laplace transform of $\cos^2(at)$, where a is a constant. (2)
    a. Do as directed.
    b. Find the Laplace transform of $f(t) = \sinh(\omega t), t \ge 0$. (3)
    c. Find the Inverse Laplace transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s-3)}$. (4)

93. Do as directed.

a. Solve the IVP using the Laplace transform $y'' + 4y = 0; y(0) = 1, y'(0) = 6$. (3)

b. Find the inverse Laplace transform of $\frac{(6+s)}{(s^2+6s+13)}$, use Shifting theorem. (4)

5.3 (May 2011)

94. Find the Laplace transform of $\left[\frac{\sin \omega t}{t}\right]$. (2)

95. Obtain $L^{-1}\left[\log \frac{1}{s}\right]$. (2)

96. Solve the simultaneous equations: Using Laplace transform $\frac{dx}{dt} - y = e^t, \frac{dy}{dt} + x = \sin t$ given $x(0) = 1, y(0) = 0$. (6)

97. Find the Laplace transform of the function $f(t) = |\sin \omega t|, t \geq 0$. (3)

98. Using Convolution theorem, find the inverse Laplace transform of $\frac{1}{(s^2+a^2)^2}$. (3)

5.4 (December 2010)

99. Find the convolution of $t \,\&\, e^t$. (2)

100. Find $L^{-1}\left[\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}\right]$. (2)

101. Using Laplace transform solve the IVP $y'' + y = \sin 2t, y(0) = 2, y'(0) = 1$. (5)

102. Find the Laplace transform of (4)

a. $t^2 \sin \pi t$.

b. $\log \frac{s+a}{s+b}$.

103. State Convolution theorem and use to evaluate $L^{-1}\left[\frac{1}{(s^2+\omega^2)^2}\right]$. (4)

104. Using Laplace transform, find the solutions of the IVP $x\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} = xt, u(x,0) = 0$; if $x \geq 0, u(0,t) = 0:\ if\ \ t \geq 0.$ (6)

5.5 (March 2010)

105. Find $L^{-1}\left[-\dfrac{s+10}{s^2-s-2}\right]$. (3)

106. Find $L^{-1}\left[\dfrac{s^3+2s^2+2}{s^3\left(s^2+1\right)}\right]$. (3)

107. State convolution theorem and use it to evaluate Laplace inverse of $\dfrac{a}{s^2\left(s^2+a^2\right)}$. (4)

108. Find the Laplace transform of half-wave rectification of $\sin\omega t$ defined by:

$$f(t)=\begin{cases}\sin\omega t; 0<t<\dfrac{\pi}{\omega}\\ 0; \dfrac{\pi}{\omega}<t<\dfrac{2\pi}{\omega}\end{cases}$$ where $f\left(t+\dfrac{2n\pi}{\omega}\right)=f(t)$ for all integer n. (3)

109. Find $L^{-1}\left[\dfrac{s^3}{s^4-81}\right]$. (3)

110. By Laplace transform solve, $y''+a^2y=K\sin at$. (4)

111. Find the inverse transform of the function $\ln\left(1+\dfrac{\omega^2}{s^2}\right)$. (3)

5.6 (December 2009)

112. Find $L\{\sin 2t\cos 2t\}$. (2)

113. By using the method of Laplace transform solve the IVP
$y''+2y'+y=e^{-t}, y(0)=-1 \,\&\, y'(0)=1$. (7)

114. Evaluate : $L^{-1}\left[\dfrac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}\right]$. (2)

115. Evaluate : $L^{-1}\left[\dfrac{3}{s^2+6s+18}\right]$. (2)

116. By using first shifting theorem, obtain the value of $L\left[(t+1)^2 e^t\right]$. (2)

117. Find the value of (4)
   a. $L\left[t\sin wt\right]$
   b. $1*1$ where $*$ denote convolution product.

118. Evaluate $L^{-1}\left[\dfrac{se^{-2s}}{s^2+\pi^2}\right]$. (3)

119. Using convolution theorem, obtain the value of $L^{-1}\left[\dfrac{1}{s\left(s^2+4\right)}\right]$. (3)

6. Series solution of ordinary Differential Equations.

6.1 (May 2012)

120. Determine the singular points of differential equation $2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$ and classify them as regular or irregular. (2)

121. Find the series solution of $(1+x^2)y'' + xy' - 9y = 0$. (4)

122. Find the series solution by using Frobenious method $xy'' + y' - y = 0$. (4)

6.2 (December 2011)

123. Determine if $x=1$ is a regular singular point of $(1-x^2)y'' - 2xy' + n(n+1)y = 0$, where n is a constant. (2)

124. Find a power series solution in powers of x of $y' + 2xy = 0$. (4)

125. Find a series solution of $y'' + y = 0$ near $x = 0$. (4)

126. Solve by Frobenious method at $x = 0 : x(x-1)y'' + (3x-1)y' + y = 0$. (4)

6.3 (May 2011)

127. Solve $y' = 2xy$ by power series method. (2)

128. Find power series solution of the equation $(1-x^2)y'' - xy' + py = 0, p$ is arbitrary constant. (4)

129. Find the series solution of $xy'' + y' + xy = 0$. (7)

130. Find the power series solution of the equation $(x^2+1)y'' + xy' - xy = 0$ about an ordinary point. (3)

6.4 (December 2010)

131. Find a series solution of he differential equation $x^2y'' + x^3y' + (x^2-2)y = 0$ by Frobenious method. (6)

6.5 (March 2010)

132. If possible find the series solution of $y'' = y'$. (3)

133. Find a series solution of differential equation $xy'' + 2y' + xy = 0$. (4)

134. Find a series solution of differential equation $(x^2 - x)y'' - xy' + y = 0$ (4)

135. Solve the Legendre's equation $(1-x^2)y'' - 2xy' + n(n+1)y = 0$ for $n = 0$. (4)

136. Obtain the Legendre's function as a solution of $(1-x^2)y'' - 2xy' + 2y = 0$. (4)

6.6 (December 2009)

137. Using the method of series solution, solve the differential equation $y'' + y = 0$. (4)

7. Fourier Series

7.1 (May 2012)

138. Find half range cosine series for $f(x) = e^x$ in $(0,1)$. (1)

139. Find Fourier series for $f(x) = \begin{cases} -\pi; -\pi \le x \le 0 \\ x; 0 \le x \le \pi \end{cases}$ (4)

and show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ...$

140. Find fourier series for $f(x) = 2x - x^2$ in the interval $(0,3)$. (4)

141. Find half range cosine series for $\sin x$ in $(0,\pi)$ and show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + ...$

and using Parseval's identity prove that $\frac{\pi^2 - 8}{16} = \frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + ...$ (3)

142. If $f(x) = \begin{cases} mx; 0 \le x \le \frac{\pi}{2} \\ m(\pi - x); \frac{\pi}{2} \le x \le \pi \end{cases}$ then show that (5)

$f(x) = \frac{4m}{\pi}\left\{\frac{\sin x}{1^2} - \frac{\sin 3x}{3^2} + \frac{\sin 5x}{5^2} - .....\right\}$.

7.2 (December 2011)

143. Find the fourier series expansions of

a. $f(x) = x; -\pi \le x \le \pi, f(x + 2\pi) = f(x)$ (3)

b. $f(x) = x^2; -2 \le x \le 2$ (4)

7.3 (May 2011)

144. Find the generalized fourier series expansion of $f(x); 0 < x < 3$ arising from the eigen function of $y'' + \lambda y = 0; 0 < x < l; y'(0) = 0, y'(l) = 0$. (4)

145. Find the fourier series for $f(x) = |s\infty nx|; -\pi < x < \pi$. (4)

146. Find half-range cosine series for $f(x) = \begin{cases} x; 0 < x < \frac{\pi}{2} \\ \pi - x; \frac{\pi}{2} < x < \pi \end{cases}$. (4)

147. Find fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}; -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}; 0 \le x \le \pi \end{cases}$. Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ... = \frac{\pi^2}{8}$. (4)

7.4 (December 2010)

148. Find the fourier cosine series of the periodic function $f(x) = x; (0 < x < L), \pi = 2L$. Also sketch $f(x)$ and its periodic extension. (5)

149. Find the fourier series of the periodic function $f(x) = x \sin \pi x, (0 < x < 1), \pi = 2L = 1$ (5)

150. Find the complex fourier series of the function $f(x) = x, (0 < x < 2\pi), p = 2L = 2\pi$. (4)

7.5 (March 2010)

151. Find the fourier series of $f(x) = x + |x|; -\pi < x < \pi$. (4)

152. Find fourier series expansion of $f(x) = \frac{x^2}{2}; -\pi < x < \pi$. (4)

153. Find fourier sine series of $f(x) = \pi - x; 0 < x < \pi$. (4)

154. Sketch the function $f(x) = x + \pi; -\pi < x < \pi$ where $f(x + 2\pi) = f(x)$ and find its fourier series. (4)

155. Find fourier cosine series for $f(x) = e^x; 0 < s < L$. (4)

7.6 (December 2009)

156. Find the fourier series of the function $f(x) = x^2; -\pi < x < \pi$. (5)

157. Obtain the fourier series of periodic function $f(x) = 2x$; $-1 < x < 2, p = 2L = 2$. (5)